

# Consistent Interactions of Yang-Mills Theory: A Review

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We reconsider the interacting theory of the Yang-Mills model in the Lagrangian form. We obtain all consistent interactions through deformations of the master equation in the antifield formalism. The results determine deformed structures of the gauge transformation.

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## I. INTRODUCTION

Dirac's work[1, 2, 3] was a pioneer in constrained systems, which were used for several applications in quantum field theory[4, 5, 6]. In this approach, we make the action in either Lagrangian or Hamiltonian forms [7, 8], while they are equivalent to each other[9]. In this way, we can obtain the Hamiltonian quantization through canonical variables such as coordinate and momentum, which involve constrained dynamics[10, 11, 12, 13, 14, 15]. The physical variables of a constrained system have gauge invariance and locally independent symmetry. Gauge transformations introduce some arbitrary time independent functions to the Hamilton's equations of motion. We can see that the canonical variables are not all independent. Hence, we need to imply some conditions for canonical variables: the first-class and second-class constraints. We must also generalize the frame to include both bosonic and fermionic variables.

BRST approach [16, 17, 18, 19] extended the local gauge symmetries in terms of BRST differential and co-/homological classes. It emerged as a replacement for the original gauge symmetry. We can construct the gauge symmetry from a nilpotent derivation. We see that the gauge action is invariant under a nilpotent symmetry, called the BRST symmetry. Replacing the gauge symmetry with the BRST symmetry introduces antifield, ghosts, and antighosts to each gauge variable [20, 21]. It presents the framework for solutions to the equations of motion [22, 23]. BRST cohomology also received some important extensions from the antifield formalism [22, 23, 24, 25, 26, 27, 28, 29]. Using the antifield formalism, we can construct consistent interactions among fields from coupling deformations of the master equation [30, 31].

In this paper, we reconsider the construction of consistent interactions of the Yang-Mills theory through all coupling deformations of the master equation. We can see that deformations stop at second-order. The resulting action identifies a deformed transformation as gauge symmetry and provides a commutator for it. The organization of this paper is as follows. Section II introduces the BRST differential and the antifield formalism. We consider the coupling deformations of the master equations in the antifield formalism. In § III, we describe the BRST transformation of the Yang-Mills theory. We obtain the deformation of the master equation and analyze its several orders. Eventually, we realize the entire gauge structure of the Yang-Mills theory. Section IV provides a conclusion.

## II. BRST FORMALISM

The *BRST differential*  $s$  is split into the Koszul-Tate resolution  $\delta$  and the exterior derivative  $\gamma$  along the gauge orbits [22, 23]:

$$s = \delta + \gamma. \quad (1)$$

The Koszul-Tate differential maintains the equation of motion (Euler-Lagrange equation).

For any  $X$  and  $Y$  with Grassmann parity  $\varepsilon_X$  and  $\varepsilon_Y$ , we have:

$$s(XY) = X(sY) + (-1)^{\varepsilon_Y}(sX)Y. \quad (2)$$

The BRST differential  $s$  is a nilpotent derivation:

$$s^2 = \delta^2 = \gamma^2 = 0. \quad (3)$$

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Moreover, the Koszul-Tate resolution  $\delta$  commutes with  $\gamma$ :

$$\gamma\delta + \delta\gamma = 0. \quad (4)$$

We denote the grading degree of  $s$  by the *ghost number* ( $\mathfrak{gh}$ ), being equal to one:

$$\mathfrak{gh}(s) = \mathfrak{gh}(\delta) = \mathfrak{gh}(\gamma) = 1, \quad (5)$$

with the following property

$$\mathfrak{gh}(XY) = \mathfrak{gh}(X) + \mathfrak{gh}(Y). \quad (6)$$

The ghost number consists of the pureghost number ( $\mathfrak{pgh}$ ) and the antighost number ( $\mathfrak{agh}$ ):

$$\mathfrak{gh}(X) = \mathfrak{pgh}(X) - \mathfrak{agh}(X). \quad (7)$$

We impose that

$$\mathfrak{pgh}(\delta) = 0, \quad \mathfrak{agh}(\delta) = -1, \quad \mathfrak{pgh}(\gamma) = 1, \quad \mathfrak{agh}(\gamma) = 0. \quad (8)$$

This means that the differential  $\delta$  reduces the antighost number by one, while it does not change the pureghost number.

### A. Antifield formalism

Let us consider the Lagrangian action

$$S_0^L[\phi^{\alpha_0}] = \int d^4x \mathcal{L}_0(\phi^{\alpha_0}, \partial_\mu \phi^{\alpha_0}). \quad (9)$$

It reads the equations of motion  $\delta S_0^L / \delta \phi^{\alpha_0} = 0$ . This has gauge symmetries as  $\delta_\varepsilon \phi^{\alpha_0} = Z_{\alpha_1}^{\alpha_0} \varepsilon^{\alpha_1}$ , where  $Z_{\alpha_1}^{\alpha_0}$  is a structure of the gauge group.

The field  $\phi^{\alpha_0}$  with ghost number zero may imply ghost  $C^{\alpha_1}$  with ghost number one, as well as the ghosts of ghost  $C^{\alpha_2}$  with ghost number two, etc:

$$C^A = \{C^{\alpha_1}, \dots, C^{\alpha_k}\}, \quad \mathfrak{gh}(C^{\alpha_k}) = k, \quad \varepsilon(C^{\alpha_k}) = k \pmod{2}. \quad (10)$$

We introduce antifield  $\phi_{\alpha_0}^*$  and antighosts  $C_A^* = \{C_{\alpha_1}^*, \dots, C_{\alpha_k}^*\}$  of opposite Grassmann parity:

$$\mathfrak{gh}(\phi_{\alpha_0}^*) = -\mathfrak{gh}(\phi^{\alpha_0}) - 1, \quad \varepsilon(\phi_{\alpha_0}^*) = \varepsilon(\phi^{\alpha_0}) + 1 \pmod{2}, \quad (11)$$

$$\mathfrak{gh}(C_{\alpha_k}^*) = -(k+1), \quad \varepsilon(C_{\alpha_k}^*) = k+1 \pmod{2}. \quad (12)$$

Therefore, we can define the gauge variables as

$$\Phi^A = \{\phi^{\alpha_0}, C^A\}, \quad \Phi_A^* = \{\phi_{\alpha_0}^*, C_A^*\}, \quad (13)$$

where  $\Phi^A$  is a set of fields including the original field, the ghost, and the ghosts of ghosts,  $\Phi_A^*$  provides the antifields definition.

The action of the BRST differential  $\bar{s}$  admits an antifield formalism  $\bar{s} \cdot = (\cdot, S)$ , where  $S$  stands for its generator and  $(\cdot, \cdot)$  is the *antibracket* defined in the space of fields  $\Phi^A$  and antifields  $\Phi_A^*$  by [28, 29]

$$(X, Y) \equiv \frac{\partial_r X}{\partial \Phi^A} \frac{\partial_l Y}{\partial \Phi_A^*} - \frac{\partial_r X}{\partial \Phi_A^*} \frac{\partial_l Y}{\partial \Phi^A}. \quad (14)$$

The nilpotent expression  $\bar{s}^2 = 0$  becomes equivalent to the master equation  $(S, S) = 0$ .

## B. Consistent interactions

We may construct a consistent interaction from  $S[\Phi^A, \Phi_A^*]$  in a deformed solution in powers of the coupling constant  $g$  [30]:

$$\begin{aligned} S \rightarrow \bar{S} &= S_0 + gS_1 + g^2S_2 + g^3S_3 \cdots \\ &= S + g \int d^4x a + g^2 \int d^4x b + g^3 \int d^4x c + \cdots \end{aligned} \quad (15)$$

of the master equation for the interacting theory

$$(\bar{S}, \bar{S}) = 0. \quad (16)$$

On substituting Eq. (15) into the master equation (16), we obtain the deformations of the master equations:

$$(S_0, S_0) = 0, \quad (17)$$

$$2(S_0, S_1) = 0, \quad (18)$$

$$2(S_0, S_2) + (S_1, S_1) = 0, \quad (19)$$

$$(S_0, S_3) + (S_1, S_2) = 0, \quad (20)$$

$$\vdots$$

while  $S_0$  reads as

$$S_0 = S_0^L + \phi_{\alpha_0}^* Z_{\alpha_1}^{\alpha_0} C^{\alpha_1} + \cdots, \quad (21)$$

where  $S_0^L = S_0[\Phi^A, \Phi_A^* = 0]$  is a free action.

We define the BRST differential  $s$  of the field theory by  $s \cdot = (\cdot, S_0)$ . Using the last definition, Eq. (17)–(20) are rewritten as:

$$s^2 = 0, \quad (22)$$

$$2sS_1 = 0, \quad (23)$$

$$2sS_2 + (S_1, S_1) = 0, \quad (24)$$

$$sS_3 + (S_1, S_2) = 0, \quad (25)$$

$$\vdots$$

We get all deformations of the master equation in the field theory.

## III. DEFORMATIONS OF MASTER EQUATION

We consider the Yang-Mills Lagrangian action involving a set of massless fields  $A_\mu^a$  as:

$$S_0^L[A_\mu^a] = -\frac{1}{4} \int d^4x F_{\mu\nu}^a F_a^{\mu\nu}, \quad (26)$$

where  $F_{\mu\nu}^a$  is the abelian field strengths defined by

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a, \quad F_a^{\mu\nu} = \eta^{\mu\alpha} \eta^{\nu\beta} \delta_{ab} F_{\alpha\beta}^b, \quad (27)$$

and  $\eta^{\mu\nu}$  is the SO(1,3) invariant flat metric in Minkowski space, and  $\delta_{ab}$  is a given symmetric invertible matrix with the following properties

$$\delta_{ab} = \delta_{(ab)}, \quad \delta^{ab} \delta_{bc} = \delta^a_c. \quad (28)$$

The equation of motion of (26) reads

$$\frac{\delta S_0^L}{\delta A_\mu^a} = \partial_\nu F_a^{\nu\mu} = 0. \quad (29)$$

This reveals an irreducible transformation:

$$\delta_\varepsilon A_\mu^a = \partial_\mu \varepsilon^a. \quad (30)$$

The action (26) is close according to an abelian algebra, and invariant under the gauge transformations (30).

The BRST transformation provides ghosts  $C^a$ , antifields  $A_a^{*\mu}$  and antighosts  $C_a^*$  [20, 21]:

$$\begin{array}{ccc} \varepsilon = 0 & A_\mu^a \xrightarrow{\delta_\varepsilon} \partial_\mu \varepsilon^a & \\ \text{gh}=0 & \downarrow \quad \searrow & \downarrow \\ \varepsilon = 1 & A_a^{*\mu} & C^a \\ \text{gh}=-1 & & \text{gh}=1 \\ \varepsilon = 0 & & C_a^* \\ & & \text{gh}=-2 \end{array} \quad (31)$$

The BRST differential  $s$  consisting of  $\delta$  and  $\gamma$  acts on  $A_\mu^a$ ,  $A_a^{*\mu}$ ,  $C^a$ , and  $C_a^*$ :

$$\delta A_\mu^a = 0, \quad \delta A_a^{*\mu} = -\frac{\delta S_0^L}{\delta A_\mu^a} = -\partial_\nu F_a^{\nu\mu}, \quad \delta C^a = 0, \quad \delta C_a^* = -\partial_\mu A_a^{*\mu}, \quad (32)$$

$$\gamma A_\mu^a = \partial_\mu C^a, \quad \gamma A_a^{*\mu} = 0, \quad \gamma C^a = 0, \quad \gamma C_a^* = 0. \quad (33)$$

The classical master equation of the action (26) has the minimal solution by

$$S_0 = S_0^L[A_\mu^a] + \int d^4x A_a^{*\mu} \partial_\mu C^a. \quad (34)$$

We will consider deformed solutions to the master equation (16) of the action (26). This comes to the minimal solution (34), when the coupling constant  $g$  vanishes.

### A. First-order deformation

We recognized that the first-order deformation satisfies (23). Here  $S_1$  is bosonic function with ghost number zero. We assume

$$S_1 = \int d^4x a, \quad (35)$$

where  $a$  is a local function.

The first-order deformation takes the local form

$$sa = (\delta a + \gamma a) = \partial_\mu j^\mu, \quad \mathbf{gh}(a) = 0, \quad \varepsilon(a) = 0, \quad (36)$$

where local current  $j^\mu$  shows the nonintegrated density of the first-order deformation related to the local cohomology of  $s$  at ghost number zero. To evaluate (36), we assume

$$a = \sum_{i=1}^I a_i, \quad \mathbf{agh}(a_i) = i, \quad \mathbf{gh}(a_i) = 0, \quad \varepsilon(a_i) = 0, \quad \forall i = 1, \dots, I, \quad (37)$$

$$j^\mu = \sum_{i=1}^I j^{(i)\mu}, \quad \mathbf{agh}(j^{(i)\mu}) = i, \quad \mathbf{gh}(j^{(i)\mu}) = 0, \quad \varepsilon(j^{(i)\mu}) = 0. \quad (38)$$

where  $j^{(k)\mu}$  are some local currents.

On substituting (37) and (38) into (36), we get

$$\sum_{i=1}^I \delta a_i + \sum_{i=1}^I \gamma a_i = \sum_{i=1}^I \partial_\mu j^{(i)\mu}, \quad \text{agh}(\delta a_i) = i - 1, \quad \text{agh}(\gamma a_i) = i. \quad (39)$$

It is decomposed into a number of antighost:

$\text{agh}(Z)$	$Z$
$I$	$\gamma a_I = \partial_\mu j^{(I)\mu},$
$I - 1$	$\delta a_I + \gamma a_{I-1} = \partial_\mu j^{(I-1)\mu},$
$k$	$\delta a_{k+1} + \gamma a_k = \partial_\mu j^{(k)\mu}, \quad k = 0, \dots, I - 2$

(40)

Although we strictly impose the first expression in (40) with positive antighost numbers vanishes:

$$\gamma a_I = 0; \quad I > 0, \quad a_I \in H^I(\gamma). \quad (41)$$

where  $H^I(\gamma)$  is the local cohomology of  $\gamma$  with pureghost number  $I$ .

Term  $a_I$  can exclusively reduce to  $\gamma$ -exact terms  $a_I = \gamma b_I$  related to a trivial definition, that states  $a_I = 0$ . This is plainly given by the second-order nilpotency of  $\gamma$ , which implies the unique solution for (41) up to  $\gamma$ -exact contributions, i. e.

$$a_I \rightarrow a_I + \gamma b_I, \quad (42)$$

$$\text{agh}(b_I) = I, \quad \text{pgh}(b_I) = I - 1, \quad \varepsilon(b_I) = 1. \quad (43)$$

So, the nontriviality of the first-order deformation  $a_I$  purposes the cohomology of the longitudinal differential  $\gamma$  at pureghost number equal to  $I$ , i. e.  $a_I \in H^I(\gamma)$ .

To solve (40), we need to provide  $H^I(\gamma)$  and  $H_I(\delta|d)$ :

$$\delta a_I = \partial_\mu m_I^\mu, \quad a_I \in H_I(\delta|d). \quad (44)$$

where  $H_I(\delta|d)$  is the local homology of the Koszul-Tate differential  $\delta$  with antighost number  $I$ .

For an irreducible situation, where gauge generators are field independent, we assume

$$H_I(\delta|d) = 0, \quad I > 2. \quad (45)$$

We then obtain

$$\gamma a_2 = 0, \quad (46)$$

$$\delta a_2 + \gamma a_1 = \partial_\mu j^{(1)\mu}, \quad (47)$$

$$\delta a_1 + \gamma a_0 = \partial_\mu j^{(0)\mu}. \quad (48)$$

This affords the first-order deformation as follows

$$a = a_0 + a_1 + a_2. \quad (49)$$

Let us consider (32) and (33). The local cohomology of  $\gamma$  at pureghost number one has a ghost  $C^a$ , while pureghost number two shows two ghosts  $C^a C^b$ , i. e.  $\{C^a\} \in H^1(\gamma)$  and  $\{C^a C^b\} \in H^2(\gamma)$ . From here, we solve (46):

$$a_2 = \frac{1}{2} C_a^* f_{bc}^a C^b C^c, \quad (50)$$

where  $f_{bc}^a$  are the structure constants and antisymmetric on indices  $bc$ :

$$f_{bc}^a = f_{[bc]}^a, \quad f_{mbc} = \delta_{am} f_{bc}^a, \quad f_{mbc} = -f_{bmc}. \quad (51)$$

We solve (47) by taking  $\delta$  from (50):

$$\delta a_2 = -\frac{1}{2}\partial_\mu(A_a^{*\mu}f_{bc}^a C^b C^c) + \gamma(A_a^{*\mu}f_{bc}^a C^b A_\mu^c)$$

This provides

$$a_1 = -A_a^{*\mu}f_{bc}^a C^b A_\mu^c, \quad j^\mu = -\frac{1}{2}A_a^{*\mu}f_{bc}^a C^b C^c. \quad (52)$$

We also solve (48) by taking  $\delta$  from  $a_1$ :

$$\delta a_1 = \partial_\nu(-F_a^{\nu\mu}f_{bc}^a C^b A_\mu^c) + \gamma(\frac{1}{2}F_a^{\nu\mu}f_{bc}^a A_\mu^b A_\mu^c) + \frac{1}{2}F_a^{\nu\mu}f_{bc}^a C^b F_{\nu\mu}^c. \quad (53)$$

The last term in (53) vanishes due to antisymmetric property (51c). We obtain

$$\delta a_1 - \gamma(\frac{1}{2}F_a^{\nu\mu}f_{bc}^a A_\mu^b A_\mu^c) = \partial_\nu(-F_a^{\nu\mu}f_{bc}^a C^b A_\mu^c). \quad (54)$$

This shows

$$a_0 = -\frac{1}{2}F_a^{\nu\mu}f_{bc}^a A_\mu^b A_\mu^c, \quad j^\mu = -F_a^{\nu\mu}f_{bc}^a C^b A_\mu^c. \quad (55)$$

Therefore, we get the first-order deformation up to antighost number two:

$$S_1 = -\frac{1}{2}\int d^4x \left( F_a^{\nu\mu}f_{bc}^a A_\mu^b A_\mu^c + 2A_a^{*\mu}f_{bc}^a C^b A_\mu^c - C_a^*f_{bc}^a C^b C^c \right). \quad (56)$$

Here the gauge generators are field independent, and are reduced to a sum of terms with antighost numbers from zero to two.

## B. Higher-order deformations

We now solve the second-order deformation of the master equation, (24). We shall assume

$$S_2 = \int d^4x b, \quad (57)$$

which takes the local form

$$\Delta + 2sb = \partial_\mu m^\mu. \quad (58)$$

We shall use (56) to compute

$$(S_1, S_1) \equiv \int d^4x \Delta.$$

This provides the following results:

$$\Delta = \Delta_0 + \Delta_1 + \Delta_2, \quad (59)$$

namely,

$$\Delta_0 \equiv -f_{e[m}^a f_{np]}^e F_a^{\alpha\beta} A_\alpha^m A_\beta^n C^p + 2f_{bcm}f_{np}^m \eta^{\alpha\rho} \eta^{\beta\mu} (\partial_\rho C^b) A_\mu^c A_\alpha^n A_\beta^p. \quad (60)$$

$$\Delta_1 \equiv -f_{e[m}^a f_{np]}^e A_a^{*\mu} C^m C^n A_\mu^p, \quad (61)$$

$$\Delta_2 \equiv -\frac{1}{3}f_{e[m}^a f_{np]}^e C_a^* C^m C^n C^p. \quad (62)$$

If we define

$$b \equiv b_0 + b_1 + b_2, \quad (63)$$

we get a set of equations

$$\Delta_2 + 2\gamma b_2 = \partial_\mu m^\mu, \quad (64)$$

$$\Delta_1 + \delta b_2 + 2\gamma b_1 = \partial_\mu m^\mu, \quad (65)$$

$$\Delta_0 + \delta b_1 + 2\gamma b_0 = \partial_\mu m^\mu. \quad (66)$$

Eqs. (62) and (64) lead to

$$\Delta_2 = 0, \quad b_2 = 0. \quad (67)$$

This implies the Jacobi identity:

$$f_{e[m}^a f_{np]}^e = 0. \quad (68)$$

We also derive

$$\Delta_1 = 0, \quad b_1 = 0. \quad (69)$$

Equation (66) gives

$$2f_{bc}^a k_{ma} f_{np}^m \eta^{\alpha\rho} \eta^{\beta\mu} (\partial_\rho C^b) A_\mu^c A_\alpha^n A_\beta^p + 2\gamma b_0 = \partial_\mu m^\mu. \quad (70)$$

We can solve it by substituting  $\gamma$  of vector fields  $A_\mu^a$  into  $\partial_\mu C^a$ :

$$2f_{bc}^a k_{ma} f_{np}^m \eta^{\alpha\rho} \eta^{\beta\mu} (\partial_\rho C^b) A_\mu^c A_\alpha^n A_\beta^p = \gamma \left( -\frac{1}{2} f_{bcm} f_{np}^m \eta^{\alpha\rho} \eta^{\beta\mu} A_\rho^b A_\mu^c A_\alpha^n A_\beta^p \right).$$

It provides

$$b_0 = -\frac{1}{4} f_{bcm} f_{np}^m \eta^{\alpha\rho} \eta^{\beta\mu} A_\rho^b A_\mu^c A_\alpha^n A_\beta^p.$$

We accordingly obtain the second-order deformation

$$S_2 = -\frac{1}{4} \int d^4x f_{bcm} f_{np}^m \eta^{\alpha\rho} \eta^{\beta\mu} A_\rho^b A_\mu^c A_\alpha^n A_\beta^p. \quad (71)$$

The Jacobi identity (68) shows

$$(S_1, S_2) = 0 \rightarrow S_3 = 0.$$

We then find out that all orders higher than second shall vanish:

$$S_k = 0, \quad \forall k \geq 3.$$

We solve the Yang-Mills action by the first- and second-order deformations:

$$S = S_0 + gS_1 + g^2S_2. \quad (72)$$

This includes the gauge structures decomposed into terms with antighost number from zero to two. We can see the part with antighost number zero in the Lagrangian forms. The antighost number one corresponds to the gauge generators. Higher antighost numbers shows the reducibility functions appearing in the ghosts of ghosts. All functions with order higher than second will vanish.

### C. Lagrangian and gauge structure

Setting  $\Phi_A^* = 0$  in (72), we read the entire Lagrangian action  $S_0^L$ :

$$\begin{aligned} S_0^L[A_\mu^a] = & -\frac{1}{4} \int d^4x F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{2} g \int d^4x F_a^{\nu\mu} f_{bc}^a A_\mu^b A_\mu^c \\ & - \frac{1}{4} g^2 \int d^4x f_{bcm} f_{np}^m \eta^{\alpha\rho} \eta^{\beta\mu} A_\rho^b A_\mu^c A_\alpha^n A_\beta^p. \end{aligned} \quad (73)$$

It determines the following action

$$S_0^L[A_\mu^a] = -\frac{1}{4} \int d^4x \mathcal{F}_{\mu\nu}^a \mathcal{F}_a^{\mu\nu}, \quad (74)$$

where  $\mathcal{F}_{\mu\nu}^a$  is the field strengths defined by

$$\mathcal{F}_{\mu\nu}^a = F_{\mu\nu}^a + g f_{bc}^a A_\mu^b A_\nu^c, \quad (75)$$

and  $f_{bc}^a$  are the structure constants of the Lie algebra.

The gauge symmetries read

$$\hat{\delta}_\varepsilon A_\mu^a = \partial_\mu \varepsilon^a - g f_{bc}^a \varepsilon^b A_\mu^c \equiv D_\mu \varepsilon^a, \quad (76)$$

which holds the following commutator:

$$[\hat{\delta}_{\varepsilon_1}, \hat{\delta}_{\varepsilon_2}] A_\mu^a = \hat{\delta}_\varepsilon A_\mu^a. \quad (77)$$

The gauge transformations remain abelian after consistent deformation. The antighost number one identifies the gauge transformations (76) by substituting  $C^a$  with gauge parameter  $\varepsilon^a$ . The antighost number two reads the commutator (77). The resulting model is a non-abelian Yang-Mills model constructed by abelian vector fields  $A_\mu^a$ .

#### IV. CONCLUSION

In this paper, we studied a consistently deforming Lagrangian action of the Yang-Mills model in the framework of the antifield formalism. We used the BRST differential to rewrite the deformations of the master equation. The analysis showed that all orders higher than two are trivial. The deformations stopped at second-order provide the consistent interactions being abelian to order  $g$ . Upon dismissing antifields, the entire gauge structures of the interacting theory is being realized.

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#### APPENDIX A: ANTIBRACKET STRUCTURE

For a function  $X(\psi)$  in a generic space, commutative or anticommutative, we state:

$$\frac{\partial_l X}{\partial \psi} = \frac{\vec{\partial}}{\partial \psi} X, \quad \frac{\partial_r X}{\partial \psi} = X \frac{\bar{\partial}}{\partial \psi}. \quad (A1)$$

The *left derivative*  $\partial_l$  is an ordinary derivative (left to right). The *right derivative*  $\partial_r$  is the derivative action from right to left.

For any  $X(\psi)$  in a generic space, we get

$$\frac{\partial_l X}{\partial \psi} = (-1)^{\varepsilon_\psi(\varepsilon_X+1)} \frac{\partial_r X}{\partial \psi}. \quad (A2)$$

Considering Eqs. (14) and (A2), it follows that

$$(X, Y) = -(-1)^{(\varepsilon_X+1)(\varepsilon_Y+1)} (Y, X).$$

Assuming  $X = Y$ , one can find

$$\frac{\partial_r X}{\partial \Phi^A} \frac{\partial_l X}{\partial \Phi_A^*} = (-1)^{(\varepsilon_X+1)(\varepsilon_X+1)} \frac{\partial_r X}{\partial \Phi_A^*} \frac{\partial_l X}{\partial \Phi^A}, \quad (A3)$$



For bosonic (commutative) and fermionic (anticommutative) variables, we have

$$(X, X) = \begin{cases} 2 \frac{\partial_r X}{\partial \Phi^A} \frac{\partial_l X}{\partial \Phi_A^*} & X \text{ is commutative,} \\ 0 & X \text{ is anticommutative.} \end{cases} \quad (\text{A4})$$

For any  $X$ , we have

$$((X, X), X) = 0, \quad \forall X. \quad (\text{A5})$$

Furthermore, the antibracket has the following properties:

$$(X, YZ) = (X, Y)Z + (-1)^{\varepsilon_Y \varepsilon_Z} (X, Z)Y, \quad (\text{A6})$$

$$(XY, Z) = X(Y, Z) + (-1)^{\varepsilon_X \varepsilon_Y} Y(X, Z), \quad (\text{A7})$$

$$\begin{aligned} ((X, Y), Z) + (-1)^{(\varepsilon_X + 1)(\varepsilon_Y + \varepsilon_Z)} ((Y, Z), X) \\ + (-1)^{(\varepsilon_Z + 1)(\varepsilon_X + \varepsilon_Y)} ((Z, X), Y) = 0. \end{aligned} \quad (\text{A8})$$

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- [1] P. A. M. Dirac, *Can. J. Math.* **2**, 129 (1950).
  - [2] P. A. M. Dirac, *Proc. Roy. Soc. Lond. A* **246**, 326 (1958).
  - [3] P. A. M. Dirac, *Lectures on Quantum Mechanics* (New York: Yeshiva University, 1964).
  - [4] J. L. Anderson and P. G. Bergmann, *Phys. Rev.* **83**, 1018 (1951).
  - [5] P. G. Bergmann and I. Goldberg, *Phys. Rev.* **98**, 531 (1955).
  - [6] S. Weinberg, *The Quantum Theory of Fields* (Cambridge: Cambridge University Press, 1995).
  - [7] M. J. Gotay and J. M. Nester, *Ann. Inst. H. Poincaré A* **30**, 129 (1979).
  - [8] M. J. Gotay and J. M. Nester, *Ann. Inst. H. Poincaré A* **32**, 1 (1980).
  - [9] C. Batlle, J. Gomis, J. M. Pons, and N. Roman-Roy, *J. Math. Phys.* **27**, 2953 (1986).
  - [10] D. M. Gitman and I. V. Tyutin, *Quantization of Fields with Constraints* (Berlin: Springer Verlag, 1990).
  - [11] J. Govaerts, *Hamiltonian Quantisation and Constrained Dynamics* (Leuven: Leuven University Press, 1991).
  - [12] A. Hanson and T. Regge and C. Teitelboim, *Constrained Hamiltonian Systems* (Rome: Accad. Naz. dei Lincei, 1976).
  - [13] L. D. Landau and E. M. Lifshitz, *Mechanics*, (trans.) J. B. Sykes and J. S. Bell (Oxford: Pergamon Press, 1976).
  - [14] E. C. G. Sudarshan and N. Mukunda, *Classical Dynamics* (New York: Wiley, 1974).
  - [15] K. Sundermeyer, *Constrained Dynamics, Lecture Notes in Physics* Vol. 169 (Berlin: Springer, 1982).
  - [16] C. Becchi, A. Rouet and R. Stora, *Phys. Lett. B* **52**, 344 (1974).
  - [17] C. Becchi, A. Rouet and R. Stora, *Commun. Math. Phys.* **42**, 127 (1975).
  - [18] C. Becchi, A. Rouet and R. Stora, *Ann. Phys. (NY)* **98** 287, (1976).
  - [19] I. V. Tyutin, *Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism* (LEBEDEV-75-39, 1975).
  - [20] F. Brandt, N. Dragon and M. Kreuzer, *Phys. Lett. B* **231**, 263 (1989).
  - [21] F. Brandt, N. Dragon and M. Kreuzer, *All Solutions of the Consistency Equations* (DESY 89-076, ITP-UH 2/89, 1989).
  - [22] J. M. L. Fisch and M. Henneaux, *Commun. Math. Phys.* **128**, 627 (1990).
  - [23] M. Henneaux, *Nucl. Phys. B (Proc. Suppl.)* **18A**, 47 (1990).
  - [24] I. A. Batalin and G. A. Vilkovisky, *Phys. Lett. B* **102**, 27 (1981).
  - [25] I. A. Batalin and G. A. Vilkovisky, *Phys. Rev. D* **28**, 2567 (1983).
  - [26] I. A. Batalin and G. A. Vilkovisky, *Phys. Rev. D* **30**, 508 (1984).
  - [27] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton: Princeton University Press, 1991).
  - [28] J. Gomis and J. Paris, *Nucl. Phys. B* **395**, 288 (1993). [arXiv:hep-th/9204065]
  - [29] J. Gomis and J. Paris and S. Samuel, *Phys. Rept.* **259**, 1 (1995). [arXiv:hep-th/9412228]
  - [30] G. Barnich and M. Henneaux, *Phys. Lett. B* **311**, 123 (1993) [arXiv:hep-th/9304057]
  - [31] M. Henneaux, *Contemp. Math.* **219**, 93 (1998). [arXiv:hep-th/9712226]